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# A methodology to estimate leakages in water distribution networks based on inlet flow data analysis

Gianfredi Mazzolani<sup>a</sup>, Luigi Berardi<sup>b</sup>, Daniele Laucelli<sup>b</sup>, Riccardo Martino<sup>c</sup>, Antonietta Simone<sup>b</sup>, Orazio Giustolisi<sup>b,\*</sup>

<sup>a</sup>Acquedotto Pugliese, Bari, 70125, Italy

<sup>b</sup>Politecnico di Bari, Dept. of Civil Engineering and Architecture, Bari, 70125, Italy

<sup>c</sup>Università di Napoli "Federico II", Dept. of Civil, Architectural and Environmental Engineering, Napoli, 80125, Italy

#### Abstract

Water loss control and reduction is a major issue in Water Distribution Network (WDN) management worldwide. In many WDNs, the infrastructure monitoring in terms of flow/pressure measurements through the network, is not implemented yet and only few data (e.g. measurement of WDN inflow) are available to estimate the current leakage rate. Cheap and easily applicable procedures are needed to estimate current water losses in WDNs aimed at understanding the actual magnitude of the phenomenon and planning interventions. This work presents a simple methodology, inspired by the analysis of WDN inflow data records collected in several real water distribution networks, which permits to assess leakages based on the seasonal fluctuation of water consumptions. The methodology is tested on two synthetic case studies based on the Apulian WDN, which hydraulic status is simulated by an advanced WDN model that includes a realistic pressure-dependent background leakage model. The analyses of case studies verifies the effectiveness of the methodology under fully controlled WDN configurations (e.g. neglecting measurement inaccuracies that might happen in real WDN and/or possible alterations of asset conditions over the analyzed period). The resulting estimates of leakages proved to be accurate under the analyzed condition, thus making the methodology promising for next applications on real WDNs.

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\* Corresponding author. Tel.: +39-080-596-3672. E-mail address: orazio.giustolisi@poliba.it Keywords: Water Distribution Networks: Water losses: Leakage estimates: Data assimilation.

#### 1. Introduction

Reducing leakages in WDNs has a huge economic impact [1] since it means to reduce the waste of water and energy resources, decrease cost of treatment and pumping, cut third party damages and, ultimately, reduce greenhouse gas emissions. All strategies and "best practices" (e.g. [2]) aimed at controlling and reducing leakages require the preliminary assessment of the current rate of real losses (i.e. leakages) in order to set possible targets for reductions, select effective technical actions (e.g. pressure control strategies and asset renewal/rehabilitation plans) and allocate economic resources. The methodologies to estimate current real losses are usually classified as top-down or bottom-up approaches (e.g. [3][4]). In top-down approaches, the measure or estimates of different components of the system water balance are used to derive leakages. For this reason, the reliability of top-down leakage estimate depends on the reliability of water consumption metering/assessment. Accordingly, the top-down leakage estimate usually refers to long analysis periods, depending on the water metering collection mode. Nowadays, data collection might take from hours (e.g., permanent automatic meter reading systems (AMR)), few weeks (e.g. for walk by Off Site Meter Reading (OMR)) or even months when few manual consumption readings per year are carried on. Therefore, top-down approaches are useful for drawing annual water balance but can be hardly useful to detect leakage increase during the year.

In bottom-up approaches, the assessment of leakages is based on the analysis of flow and/or pressure data monitored through the WDN. For this reason such approaches are considered more "data hungry" and time consuming, since they require the most accurate and up-to-date data as possible. The analysis of the Minimum Night Flow (MNF) is probably the most adopted bottom-up technique worldwide, permitting to estimate real losses by subtracting the expected legitimate water consumption from the recorded system inflow (e.g. [2][5]). Indeed, MNF permits to verify/integrate water balance and is the only viable option when no customers' consumption metering is available. Actually, the MNF leakage estimate has been used in conjunction with the Fixed and Variable Area Discharge (FAVAD) concept [6][7] in order to take into account the pressure-leakage dependence. Nonetheless, this poses the need for monitoring pressure through the WDN, besides collecting water inflow observations.

A different bottom-up methodology proposed by Buchberger and Nadimpalli [8] exploited the statistical analysis of flow data to estimate water losses. Besides the originality of the proposed approach, the main drawback of such methodology stems from the need of high-resolution flow data sampling (10, 5 or even 1 second), which is not technically affordable by the "smart" meters, increasingly adopted by water utilities, that transmit data sampled every 10-30 minutes using long-life batteries. In addition storing such large amount of data would be not justified by other WDN management purposes.

Unfortunately, in many WDNs worldwide, the only available information is the water flow sampled at WDN inlet points, while no flow/pressure gauges are installed within the network. In such circumstances, top-down approaches permits to estimate Non Revenue Water (NRW), based on the difference between WDN inflow data records and the authorized (and billed) water consumption, that includes both apparent losses and real water outflows (i.e. leakages). Moreover, these WDNs show high NRW rates (even higher than 50% of total inlet water volume) and the reliable estimate of current leakage rate would support prioritizing the allocation of resources for rehabilitation/renewal works and/or for implementing higher resolution flow/pressure monitoring systems within the WDN.

This contribution proposes a bottom-up methodology for leakage assessment, where the parameters of a hydraulic consistent model are estimated using WDN inflow data only, following a data assimilation [9] approach. Differently from other methodologies, the analysis can be carried on off-line, thus not requiring real-time transmission of data streams, and the sampling interval can range from few minutes to one-hour, thus being readily usable in most WDNs. Results of leakage assessment are immediately verified using recorded data, while the methodology permits to easily update leakage assessment (on-line) as soon as up-to-date data are available. In addition, the comparison of leakage estimates relating to different periods (e.g. previous years) can be used to detect the increase of leakage rate and/or verify the effects of leakage reduction actions. As such, the proposed methodology lend itself to verify/control other leakage assessment methods based on either top-down or bottom-up approaches.

# 2. Methodology

## 2.1. Background

The methodology presented herein was inspired by the analysis of water inflow data recorded in some WDNs serving urban areas in Apulia region (Southern Italy). The analyzed WDNs are characterized by mainly residential (household) consumptions with seasonal increase during summer (i.e. peak of consumptions in July-August). The water utility recorded inflow readings with 10 minutes sampling interval from 00:00 to 23:50, over one year. Figure 1 reports the average daily network inflow  $Q_d$  from 00:00 to 23:50, and the average network night inflow  $Q_{N,d}$  from 02:00 to 04:00 a.m., for each day of the year. It is evident that average night inflow  $Q_{N,d}$  follows the same trend as the average daily inflow  $Q_d$ , with summer peak due to the seasonal increase of residential population as well as to the increase of the per-capita water consumption. This observation is consistent with the fact that such WDNs usually serve small towns where residential water consumption prevails and water usage daily pattern does not sensibly change over the year. Nonetheless, Figure 1 shows that the trend of the ratio between night and daily average inflow  $(O_N / O_d)$  is not constant over the year but decreases during the peak season. This observation suggests that such decrease is due to a water outflow component that no longer relates on human water requests (i.e. proportional to the number of people and per capita water consumption), but is due to leakages, which reflect the change in WDN hydraulic functioning. Accordingly, the idea behind the proposed methodology is to exploit the seasonal variation of WDN inflow to assess WDN leakages. It is worth to remark that this methodology returns the estimate of real water outflows (leakages) only that depend on actual WDN hydraulics, without including apparent losses that are related to actual water consumptions.

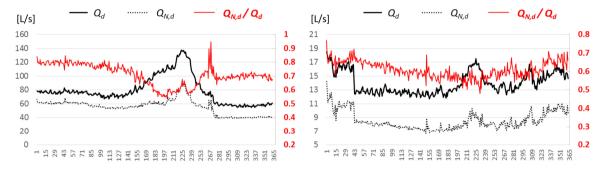


Fig. 1. Average daily  $(Q_d)$  and night  $(Q_{N,d})$  water inflow observed in two real WDNs in Apulia region (Italy).

## 2.2. Methodology assumptions

Following the background observations reported above, the present methodology requires some key hypotheses to verify in order to be applicable on real WDNs.

- 1. The ratio (i.e. *K*) between night and day water customers' requests is invariant over all days of the analyzed period, including the peak season (when population and/or per capita water consumption increase due to temperature during the summer increases). Such hypothesis holds when household water consumption prevails over others (e.g. industrial, commercial or business) and/or non-domestic water consumptions either are measured or reliably assessed, thus they can be deducted from the total WDN inflow. Accordingly, days with different day/night water usage (e.g. weekend and working days) are likely to have different values of *K*.
- 2. Over the analyzed days, sharing the same water daily usage pattern (i.e. *K*) new leakages/bursts are assumed to not happen and no repair/rehabilitation works are carried on. In the opposite case, the change in leakages over the analyzed days would bias data assimilation strategy underlying the methodology.
- 3. The time series of network water inflow from water source(s) (e.g., tanks, reservoirs, connections to other networks) should be available as a discrete sequence of values with a constant sampling time interval  $\Delta t$ .

Under such hypotheses, the following equation holds for m days showing the same daily water usage (i.e. K):

$$\frac{Q_{N,d} - Q_{LN,d}}{Q_d - Q_{L,d}} = K \qquad \rightarrow \qquad \frac{V_{N,d} - L_{N,d}}{V_d - L_d} = K \qquad \text{with} \qquad d = 1, K, m$$
 (1)

The first of Equations 1 represents the ratio between average daily and night customers' water consumption, which is assumed invariant over m days (see hypothesis 1 above). Indeed,  $Q_{N,d}$  and  $Q_d$  are the average water inflow values recorded overnight (e.g. 2:00-4:00 a.m.) and over the entire day, respectively.  $Q_{LN,d}$  and  $Q_{L,d}$  are the average leakage flow during night (e.g. 2:00-4:00 a.m.) and over the entire day, respectively. The last Equation 1, obtained by simple manipulation, reports the same relationship in terms of average water volumes over  $\Delta t$ .  $V_d$  and  $V_{N,d}$  are the daily and night average (recorded) inlet volumes over each sampling interval  $\Delta t$ , respectively.  $L_d$  and  $L_{N,d}$  are the daily and the night average leakage volume over  $\Delta t$ , respectively. Indeed using volumes permits to easily assess total volumes over multiple hours, which are of direct relevance for water utilities, and permits to use the methodology when cumulated volume data are available only (e.g. in small WDN or at inlet points of districts metering areas in large systems).

Actually, leakage outflow is known to change over the day with pressure (e.g. [6][7]) meaning that during the night the average leakage volume is higher than over the whole day, because of higher night pressure due to lower water usage. Nonetheless, the average night leakage volume  $L_{N,d}$  can be assumed as invariant among the m days, since the small differences in night water consumptions are likely to not sensibly affect pressure regime, and thus it is designated a  $L_N$ . Accordingly, the daily average leakage volume  $L_d$  can be formulated as a fraction of  $L_N$  and the Eq. (1) can be written as:

$$\frac{V_{N,d} - L_N}{V_d - [a_d] \cdot L_N} = K \qquad \text{with} \quad [a_d] \le 1 \quad \text{and} \quad d = 1, K, m$$
 (2)

The term  $[a_d]$  entails the effect of pressure on leakages, depending on daily variation of customers' water usage and WDN asset conditions. The equality  $[a_d]=1$  strictly holds for those WDN with roughly invariant pressure over the day. This happens, for example, in WDNs that are oversized with respect to normal operating conditions (e.g. to guarantee sufficient pressure also under large water firefighting demand) or WDNs that are subject to "smart" pressure control (e.g. by a Remotely Real-Time Controlled Pressure Control Valves, RRTC PCV, maintaining a desired pressure value at a "critical" node within the WDN (e.g. [10]).

Using a monomial pressure-leakage expression like  $Q_L = \beta P^{\gamma}$  (e.g. [11]), the expression for  $[a_d]$  can be written as:

$$\left[a_{d}\right] = \frac{L_{d}}{L_{N}} = \frac{\beta P_{d}^{\gamma}}{\beta P_{N}^{\gamma}} = \left(\frac{P_{d}}{P_{N}}\right)^{\gamma} \tag{3}$$

where  $P_N$  is the night average pressure over the m days that is related to the "invariant" night average leakage volume  $L_N$ , and  $P_d$  is the average daily pressure over the d-th day related to the average daily leakage volume  $L_d$ . Nonetheless, when pressure measurements are not available, relationships like Equation 3 cannot be used and  $[a_d]$  should be assessed based on inflow records only, i.e. through data assimilation techniques.

## 2.3. Methodology formulation

Data assimilation is "an analysis technique in which the observed information is accumulated into the model state by taking advantage of consistency constraints with laws of time evolution and physical properties" [9]. In WDN leakage assessment, the collection of inlet volume data (i.e.  $V_d$  and  $V_{N,d}$ ) represents the observation of the true state of the system, while the consistency with WDN physical background is given by the main assumptions that are formulated as in Equation 2. For the sake of explanation, let us consider m days (out of 365 days of the year) where K is expected to be invariant (e.g. m working days). Based on the remarks reported in the previous section, K and  $L_N$  are system's invariant parameters that can be estimated by solving the following system of non-linear equations using

inlet water volume data series of the m days:

$$\begin{cases} K \cdot V_{d=1} - K \cdot [a_{d=1}] \cdot L_N + L_N = V_{N,d=1} \\ K \cdot V_{d=2} - K \cdot [a_{d=2}] \cdot L_N + L_N = V_{N,d=2} \\ M \\ K \cdot V_{d=m} - K \cdot [a_{d=m}] \cdot L_N + L_N = V_{N,d=m} \end{cases}$$
(4)

Several numerical methods exist to solve the non-linear system (4) (e.g. [12]) and the robustness of K and  $L_N$  in the system (4) increases with the number of independent analyzed days. In addition, in order to reduce the risk of biasing the estimates of K and  $L_N$ , the m days should range between low water consumption season (e.g. winter-autumn) to peak season (e.g. summer). Overall, the system (4) can be written for different sets of days entailing different water usage patterns (i.e. different K) based on prior assumption of the analyzer. For instance, working days (e.g. from Monday to Friday) are expected to have a lower value of K than weekend or vacation days when water usage during night is expected to be higher. The analysis of the estimates also permits to verify a posteriori the relevance of prior selection of the m days to apply the methodology. About the term  $[a_d]$  in system (4), the proposed methodology permits to introduce various expressions, each related to a different formulation of the expected physical WDN behavior, if pressure data are not available. This contribution proposes three alternative formulations as reported in the following Eqs (5).

$$[a_d]; 1 \longrightarrow K \cdot V_d - K \cdot L_N + L_N = V_{N,d}$$
 (A)

$$[a_d]; \left(\frac{V_N^{\text{avg}}}{V_d}\right)^{\alpha} \longrightarrow K \cdot V_d - K \cdot \left(\frac{V_N^{\text{avg}}}{V_d}\right)^{\alpha} \cdot L_N + L_N = V_{N,d}$$
(B)

$$[a_d]; 1-b\left(\frac{V_d}{V_N^{avg}}\right)^{\delta} \rightarrow K \cdot V_d - K \cdot \left[1-b\left(\frac{V_d}{V_N^{avg}}\right)^{\delta}\right] \cdot L_N + L_N = V_{N,d}$$
 (C)

Formulation (A) holds where the effect of pressure variation over the day is roughly negligible, as above discussed. Formulation (B) is consistent with the common hydraulic WDN behavior where the higher average daily water inflow (i.e. volume  $V_d$  in  $\Delta t$ ), the lower the average daily pressure (i.e.  $P_d$  related to  $L_d$ ). Similarly, the higher the night inlet volume averaged over the m days ( $V_N^{org}$ ) with respect to  $V_d$ , the lower the average night pressure over the m days (i.e.  $P_N$  as related to the "invariant" night average leakage volume  $L_N$ ). It is worth noting that, in general, the exponent  $\alpha > 0$  is different from  $\gamma$  in Eq. (3), since the relationship between average WDN pressure and total inlet volume is not linear. Formulation (C) is still consistent with the hydraulic condition that  $L_d < L_N$ , while encompassing also the case of  $L_d = L_N$ , that was possible in formulation (B) only for  $\alpha \to 0$  because  $V_N V_d < 1$  always. Large values of coefficient b are likely to represent WDNs with large pressure (and leakage) variations over the day.  $V_i = 0$ 0, the effect of pressure variations is negligible, as reported in Formulation (A) (i.e.  $a_d$  tends to 1). In addition, exponent  $\delta$  is different from both  $\gamma$  in Eq. (3) and  $\alpha$  in (B).

# 3. Tests on Apulian WDN

This contribution reports the tests of the methodology using two synthetic case studies based on the Apulian WDN whose layout is in Figure 2(a) (for details see [13]). The hydraulic status of the WDN including flows, pressures and leakages are simulated using the WDNetXL system [14] since it allows the realistic pressure-driven analysis of all water demand components, including pressure-driven background leakages as outflows distributed along pipes ([15] [16] [17]). Using such model, in turn, permitted to modify the original Apulian WDN, in order to simulate different hydraulic conditions. Figure (2b) reports the daily base demand pattern used for all simulations, showing the minimum night flow hours as white triangles at 2.00-4.00 a.m. Figure (2c) shows the multipliers of the base pattern that is used to emulate the seasonal variation of water requests. In order to account for realistic water consumption variations among the days of the year, seasonal multipliers are randomly sampled in a range of  $\pm 10\%$  of the mean value. For the sake of the example, the ratio K is assumed the same for

all days, thus neglecting the differences between weekdays and working days.

Fig. 2. (a) Apulian network layout; (b) base daily demand pattern; (c) seasonal multiplying factors.

The methodology is tested using the following two WDN configurations:

(Case 1) Pressure invariant scenario. The original customers' base demands [13] are multiplied by a factor 0.1, so that the network is oversized with respect to water requests and a RRTC-PCV is assumed on pipe 34, controlling pressure at node 13 with pressure set at 15m. The parameters of the background leakage model [17] were estimated in order to have, under such pressure control scenario, a leakage rate of 22% of total inlet water volume over 365 days extended period simulation (EPS). Due to pressure control, the maximum pressure variation over the day is less than 1m

(Case 2) Pressure variation scenario. In order to emphasize the change in pressure, the customers' base demands of case 1 are multiplied by a factor 3 and the elevation of reservoir 24 was raised to 50m, without pressure control valves. Leakage parameters were estimated to get a leakage rate of about 30%. This results into maximum pressure variation between night and day of about 20m during summer.

Figure (3) and Table 1 summarize the results of leakage analyses. The top graph reports, all key volume figures for each day of the year, while the two bottom graphs in blue and black squares report the zoom on total daily leakages volume (24 hours) and night leakage volume (from 2.00 to 4.00 a.m.) respectively. In *case 1* the daily and night leakage volume ( $V_{leak}$ ) are constant over 365 days, due to pressure control by the RRTC-PRV. Using formulation (5-A) the methodology returns exactly the known leakage rate (i.e. 22%) and value of K (see Table 1). Although parameters  $\alpha$ ,  $\delta$  and b estimated for formulations (5-B) and (5-C) are non-null, the error on leakage rate is lower than 2% of the actual value. In *case 2*, where pressure variations is emphasized, the leakage volume obtained with formulation (C) (i.e.  $V_{leak}(C)$ ) clearly follows the same trend of  $V_{leak}$ , with about 0.7% of leakage rate overestimation, which is likely due to the hypothesis of invariant  $L_N$ . The estimate of exponent  $\alpha$  is close to zero, thus making  $V_{leak}(B) \approx V_{leak}(A)$  and suggesting that the simple expression (5-B) for  $[a_d]$  does not permit to follow large pressure changes.

	"known" values		Form.	Leakage	K	$L_N$	$\alpha$ or $\delta$	b
	Leakage rate [%]	K	— FOIIII.	rate [%]	Λ	$[m^3/h]$	α 01 <i>0</i>	b
Case 1	22.1	0.154	(5-A)	22.0	0.154	18.2	-	-
			(5-B)	21.7	0.153	18.2	1.48×10-2	-
			(5-C)	21.9	0.153	18.1	6.38×10-1	0.001
Case 2	29.3	0.154	(5-A)	32.8	0.162	89.8	-	-
			(5-B)	32.8	0.162	89.8	5.11×10-14	-
			(5-C)	30.0	0.150	90.8	2.19	0.015

Table 1. Leakage assessment results.

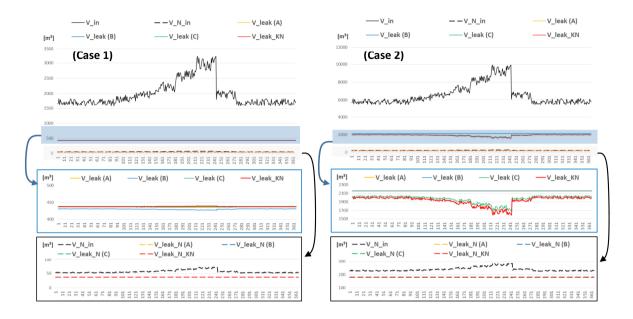


Fig. 3. Leakage assessment results: Case 1 (left); Case 2(right).

The comparison between the two cases, suggests that, when the three formulations return almost the same estimate of the leakage rate, even with non-null parameters (i.e.  $\alpha$ ,  $\delta$  and b), the effect of pressure changes during the peak season is low and the estimate of leakages is reliable. In the opposite case, when pressure changes affect daily leakages, formulation (5-B) tends to fall into (5-A), while results for (5-C) are consistent with the WDN hydraulic behavior.

#### 4. Conclusions

This paper presents a bottom-up methodology for leakage assessment in WDNs, based on a physically consistent formulation of the problem and using WDN inflow readings only in a data assimilation approach. The tests performed on two synthetic WDNs, where the main assumptions of the methodology are verified, show that the procedure is able to return reliable estimates of current leakage rate under different hydraulic conditions. The comparison of results obtained by assuming different formulations for pressure-dependent term  $[a_d]$  provides a further criterion for analyzing results. Results showed herein have motivated ongoing research on application of the proposed methodology on real WDNs aimed at investigating alternative formulations for term  $[a_d]$  as well as criteria to select the most reliable leakage rate prediction. It is worth noting that, differently from other bottom-up approaches, the proposed methodology permits to assess the night users' water consumption together with leakages using inlet volume records only; thus proving useful when water metering are not available to validate other methods.

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